International Journal of Advanced Educational Research
ISSN: 2455-6157
Impact Factor: RJIF 5.12
www.educationjournal.org
Volume 2; Issue 5; September 2017; Page No. 295-298

Symmetries lead to conservation principles

[1] Tamanna Punia, [2] Attar Singh
[1] Arya College of Education, CRS Univ. Jind, Hisar, Haryana, India
[2] Assoc. Prof. HOD Physics, CRM PG College, Hisar, Haryana, India

Abstract
To discover and understand the order in nature, symmetry considerations provide an extremely powerful and useful tool. Until Einstein's special theory of relativity came into picture, it was considered natural to obtain "conservation principles" from 'laws of nature'. Afterwards Emmy Noether with his work concluded by first examining the symmetry considerations that have conservation principles associated with them, laws of nature are deduced from the underlying connections between conservation laws and symmetry. Gradually, symmetries have become the backbone of our theoretical formulation of the physical laws. The important result of the existence of symmetry (or invariance) is that a corresponding conservation is followed. In this paper, we shall explain this statement and its results, mainly within the framework of contemporary physics.

Keywords: invariance, particle physics, symmetry, transformations, physical laws, fundamental interactions

Introduction

Symmetry considerations have provided us an extremely strong and useful tool in our effort to understand nature since the beginning of physics. They have become the backbone of our theoretical formulation of physical laws moderately. Principle of symmetry plays a major role in Physics; it is a well-known fact. A significant theorem discovered early in this century by Emmy Noether, the German mathematician states that "Every conservation principle corresponds to symmetry in nature". But, what is meant by "symmetry"? Generally, symmetry of a specific kind exists when a certain operation leaves something same (unchanged). For example, a uniform cylindrical rod is symmetric with respect to reflection in a mirror; it is also symmetric about a vertical axis because it can be rotated about that axis without changing in appearance or any other feature.

The simplest symmetry operation is translation in space, which means that laws of physics are independent on choice of origin of our coordinate system. Noether showed that the invariance of the description of nature to translation in space results in the conservation of linear momentum. Another simple symmetry operation is translation in time, which means that the laws of physics are independent of when we choose t=0 and this invariance leads to conservation of energy. When laws of physics don't depend on the orientation of the coordinate system in which they are expressed leads to invariance under rotations in space and hence conservation of angular momentum.

Gauge transformation is the reason of conservation of electric charge. Gauge transformations are basically shifts in the zeros of the scalar and vector electromagnetic potentials V and A. (According to electromagnetic theory, the electromagnetic field can be explained in terms of potentials V and A instead of in terms of E and B, where the two are related by vector calculus formulas:

$E = -\nabla V$ and $B = \nabla \times A$

Both E and B remain unchanged under Gauge transformation since they are obtained by differentiating the potentials and hence this leads to charge conservation. Symmetries under reflection in space and under exchange of particle and antiparticle correspond to conservation of *parity (P)* and *charge-conjugation (C)*, respectively.

The approximate symmetry of up (u) and down (d) quarks leads to the conservation of isospin, and so forth. If we think about what symmetry means in such cases as above, we realize that it refers to a situation in which there exist several distinguish states that, however, are basically equivalent to each other with respect to the laws of physics; symmetry imply that the laws of physics are invariant to the symmetry operations in which a state is replaced by an equivalent state.

There is a need to differentiate between two equivalent states in such cases. In Quantum Mechanics such states are represented by: /X> to /X´> and the quantities used in such expressions are associated with the conserved quantities, or quantum numbers. Existence of an arbitrary state /X> would imply that any equivalent state formed by a symmetry operation must also be possible, if physical laws possess a symmetry. An illustration of above is:

An electron with spin with spin component $S_z = + \hbar/2$ can be placed in a magnetic field along $\hat{z}$, as well as rotated field along $\hat{z}'$. Both states are possible and the characteristics of the electron don't depend on the direction in principle.

Groups of Symmetries

There are four main groups of symmetries that are found to be useful and important in physics:

a. **Permutation Symmetry:** Bose-Einstein and Fermi-Dirac statistics. The interchange of identical particles in a system is a type of symmetry operation which leads to preservation of the character of the wave function of a system. The wave function may be symmetric under such as interchange, in which case the particles don't obey the exclusion principle and the system follows Bose-Einstein statistics, or it may be antisymmetric, in which case the particles obey the exclusion principle and the system follows Fermi-Dirac statistics.

b. Discrete Symmetry: A discrete symmetry is a symmetry that describes non-continous changes in a system. These include space inversion, particle-antiparticle conjugation, time reversal etc.

c. Unitary(internal) symmetries: These include:

a) U(1) – symmetries, such as those related to the conservation of baryon number, lepton number, electric charge etc.
b) SU(2) (isospin) - symmetry
c) SU(3) (color) – symmetry
d) SU(n) (flavor) – symmetry

Among these U(1) symmetry and SU(3) (color) – symmetry are believed to be exact. Rest of them seems to be broken.

d. Continuous Space-time symmetries: These spatial symmetries are those where a property of the system doesn't change with a continuous change in location. These include translation, rotation, acceleration etc.

Symmetries and Conservation laws; Noether's Theorem

The assumption that it is impossible to observe certain fundamental quantities (these are called "non- observable") is the root of all symmetry principles.

Let us show the relation among symmetry transformations, conservation laws and non – observables by simple examples.

Suppose there are two particles at position $\vec{r}_1$ and $\vec{r}_2$ and the interaction energy between the two particles is V. Absolute position is a non-observable this assumption means that we can arbitrarily choose the origin O from which these position vectors are drawn; the interaction energy V should be independent of O. In other words, we can say that V is invariant under arbitrary space translation, changing O to any other point O´.

$$\vec{r}_1 \to \vec{r}_1 + \vec{d} \text{ and } \vec{r}_2 \to \vec{r}_2 + \vec{d} \quad (1)$$

As a result, V is a function only of the relative distance $|r1 - r2|$:

$$V = V(|\vec{r}_1 - \vec{r}_2|) \quad (2)$$

And we know that for one particle, $\mathbf{F} = -\vec{\nabla} V$

So, from this, we deduce that the total momentum of this system of two particles must be conserved because its rate of change is equal to the force

$$F = -\left(\vec{\nabla}_1 + \vec{\nabla}_2\right)V$$

which is zero on account of (2).

The interdependence among three aspects of a symmetry principle:

a. The implied invariance under the connected mathematical transformation,
b. The physical existence of a non- observable, and
c. The physical results of a conservation law, which can be stated as a selection rule are shown with the help of this example.

Similarly, we may assume the absolute time to be a non-observable. Then under a time translation, the physical law must be invariant.

$$t \to t + \tau$$

As a result, there is a conservation of energy. We can derive rotation invariance and hence obtain the conservation law of angular momentum by assuming the absolute spatial direction to be non- observable. Similarly, we can derive the symmetry requirement of Lorentz invariance, and with it the conservation laws connected with the six generators of Lorentz group by assuming that absolute (uniform) velocity is not an observable. Similarly, the foundation of general relativity rests on the assumption that it is impossible to differentiate the variations between acceleration and a suitably arranged gravitation field.

Therefore, invariance under time displacement, translations, rotations, and Lorentz transformations leads to conservation of energy, momentum and angular momentum. Besides these conservation laws, associated with "external" symmetry transformations that don't mix fields with different space-time properties, i.e. transformations that commute with the space- time components of the wave function.

An electron is described by an array of complex fields (Ψ) as an example and the corresponding Lagrangian can be written as:

$$L = \frac{i}{2}\left[\overline{\psi}\gamma_\mu \partial^\mu \psi - \left(\partial^\mu \overline{\psi}\right)\gamma_\mu \psi\right] - m\overline{\psi}\psi \quad (3)$$

Under the phase transformation, this Lagrangian (3) is invariant.

$$\psi \to e^{i\alpha}\psi, \text{ where } \alpha \text{ is a real constant.} \quad (4)$$

By noting that, $\partial_\mu \psi \to e^{i\alpha}\partial_\mu \psi$ and $\overline{\psi} \to e^{-i\alpha}\overline{\psi}$, this can be easily checked.

The U(1) group, also known as unitary Abelian group is formed by the family of phase transformation

$U(\alpha) \equiv e^{i\alpha}$, where a single parameter α can have any real values.

It is recorded that group multiplication is commutative.

$$U(\alpha_1)\,U(\alpha_2) = U(\alpha_2)\,U(\alpha_1) \quad (5)$$

Through Noether's theorem, it implies the existence of a conserved current. To observe this, it is sufficient to study the invariance of α under an infinitesimal U (1) transformation.

$\psi \rightarrow (1+i\alpha)\psi.$ (6)

As we know, it is required that lagrangian remains unchanged in invariance i.e.

$$\delta L = 0$$

$$\Rightarrow \frac{\partial L}{\partial \psi}\delta\psi + \frac{\partial L}{\partial(\partial_\mu \psi)}\delta(\partial_\mu \psi) + \delta\overline{\psi}\frac{\partial L}{\partial \overline{\psi}} + \delta(\partial_\mu \overline{\psi})\frac{\partial L}{\partial(\partial_\mu \overline{\psi})}$$

$$\Rightarrow \frac{\partial L}{\partial \psi}(i\alpha\psi) + \frac{\partial L}{\partial(\partial_\mu \psi)}(i\alpha\partial_\mu \psi) + \ldots \quad (7)$$

$$\Rightarrow i\alpha\left[\frac{\partial L}{\partial \psi} - \partial_\mu \frac{\partial L}{\partial(\partial_\mu \psi)}\right]\psi + i\alpha\partial_\mu\left(\frac{\partial L}{\partial(\partial_\mu \psi)}\psi\right) + \ldots$$

The term in square brackets becomes zero by virtue of the Euler-Lagrange equation for ψ, i.e.

$$\frac{\partial}{\partial x_\mu}\left(\frac{\partial L}{\partial(\partial\psi/\partial x_\mu)}\right) - \frac{\partial L}{\partial \psi} = 0 \quad (8)$$

(and similarly for ψ), and so (7) reduces to the form of an equation for a conserved Current:

$$\partial_\mu j^\mu = 0 \quad (9)$$

Where

$$j^\mu = \frac{ie}{2}\left(\frac{\partial L}{\partial(\partial_\mu \psi)}\psi - \overline{\psi}\frac{\partial L}{\partial(\partial_\mu \overline{\psi})}\right) = -e\overline{\psi}\gamma^\mu\psi, \quad (10)$$

To match up with the electromagnetic charge current density of an electron of charge *–e*, the proportionality factor is chosen. Because of the *U*(1) phase invariance, the total charge

$$Q = \int j^0 \, d^3x \quad (11)$$

Must be a conserved quantity (followed from equation (9). Thus, the mathematical expression that relates the existence of conserved charges (*Q*) and the existence of symmetries has been obtained - a result that has intense impact in physics. We summarize these three fundamental aspects for some of the symmetry principles used in physics in table 1.

Table 1: Summary of fundamental aspects for some of the symmetry principles used in physics

Non-observables	Symmetry transformation	Conservation of Laws or Selection Rules
Absolute right (or absolute left)	$\vec{r} \rightarrow -\vec{r}$	Parity
Absolute sign of electric charge	e → -e (or $\psi \rightarrow e^{i\alpha}(\overline{\psi})'$)	Charge conjugation
Absolute spatial direction	Rotation $\hat{r} \rightarrow \hat{r}'$	Angular momentum
Absolute spatial position	Space translation $\vec{r} \rightarrow \vec{r} + \vec{d}$	Momentum
Absolute time	time translation $t \rightarrow t + \tau$	Energy
Difference between different coherent mixture of p and n states	$\begin{pmatrix} p \\ n \end{pmatrix} \rightarrow u \begin{pmatrix} p \\ n \end{pmatrix}$	Isospin
Difference between identical particles	Permutation	Bose-Einstein or Fermi-Dirac Statistics
Relative phase between states of different baryon number N	$\psi \rightarrow e^{iN\theta}\psi$	Baryon number
Relative phase between states of different charge Q	$\psi \rightarrow e^{iQ\theta}\psi$	Charge
Relative phase between states of different lepton number L	$\psi \rightarrow e^{iL\theta}\psi$	Lepton number

Conclusion

Symmetry considerations provide an extremely powerful and useful tool in our efforts to discover and understand the order in nature. This paper has described when a physical system is invariant under certain transformations and how these symmetries correspond to a particular conserved quantity. We have also seen how rotational symmetry and the conservation of angular momentum are two sides of the same coin and time translation symmetry and energy conservation are related and many more symmetries have also been discussed.

References

1. Hooft G. Nobel Lecture: A confrontation with infinity, Review of Modern Physics. 2000; 72:333-339. [This article presents a brilliant exposition of the work of the co-winner of 1999 Nobel Prize in physics, the author of the proof that Yang-Mills theories with spontaneous symmetry breaking are renormalizable. In this paper it is also discussed the perspectives for the future of high energy physics].
2. Abers ES, Lee BW. *Gauge Theories*, Physics Reports. 1973; 9:1-141. [This report offers an excellent review of gauge theories].
3. Cheng TP, Li LF. Gauge Theory of Elementary Particle Physics, 536 pp. Oxford Science Publications, Oxford, U.K. [This book is an excellent introduction to the

foundation of gauge theories of electroweak and strong interactions].
4. Gell-Mann M. A Schematic Model of Baryons and Mesons, Physics Letters. 1964; 8:214-215. [This is a pioneering work in which the author proposed the existence of quarks].
5. Goldstein H. *Classic Mechanics*, Second Edition, Addison-Wesley, 1980.
6. Halzen F, Martin AD. Quarks and Leptons: An Introductory Course in Modern Particle Physics, 396. John Wiley & Sons, Inc. New York. [This book offers a pedagogical introduction to the physics of elementary particles].
7. Hanca J, Taylor EF. From conservation of energy to the principle of least action: A story line', Am. J Phys. 2004'; 72(4):514-521.
8. Hanca J, Taylor EF, Tulejac S. `Deriving Lagrange's equations using elementary calculus', Am. J Phys. 2004; 72(4):510-513.
9. Huang K. Quarks, Leptons and Gauge Fields. 1992; 333. World Scientific Publishing Co.,
10. Riley KF, Hobson MP, Bence SJ. Mathematical Methods for Physics and Engineering. Cambridge University Press, third edition, 2006.
11. Lee TD. Particle Physics and Introduction to Field Theory, 865. Harwood Academic Publishers GmbH, Chur, Switzerland. [This book presents a comprehensive discussion to the ideas that have shaped our thinking about the elementary constituents of matter].
12. Moore Thomas A. Getting the most action out of least action: A proposal', Am. J Phys. 2004; 72(4):522-527.
13. Nambu Y. *Quarks*, World Scientific Publishing Co., Singapore. 1985; 228. [This book presents a comprehensive historical development of the ideas which helped to influence our current perception of elementary particles].
14. Deshmukh PC, Libby J. Symmetry Principles and Conservation Laws in Atomic and Subatomic Physics, Resonance. 2010; 926: 832.
15. Singapore. [This book presents a concise introduction to the physical motivation behind symmetries and a precise mathematical formulation thereof].
16. Kibble TWB. Classical Mechanics. McGraw-Hill, second edition, 1973.
17. University Press, Cambridge UK. [This book is an excellent and pedagogical introduction to the understanding of the foundations of quantum field theory].
18. Veltman MJG. Nobel Lecture: From weak interactions to gravitation, Review of Modern Physics. 2000; 72:341-349. [In this article the co-winner of 1999 Nobel Prize presents his personal view of the path that led him to work for almost a decade in the foundation of renormalization theory, as a necessary step in order to understand weak interactions].
19. Weinberg S. The Quantum Theory of Fields, Volume I: Foundations, Cambridge, 1995; 609.
20. Weinberg S. The Quantum Theory of Fields, Volume II: Modern Applications, Cambridge University Press, Cambridge, U.K. 1996, 489. [This book is an excellent and pedagogical introduction to the understanding of the applications of quantum field theory].
21. Weinberg S. The Quantum Theory of Fields, Volume III: Super symmetry, Cambridge University Press, Cambridge, U.K. 2000, 419. [This book is an excellent and pedagogical introduction to the understanding of the application of quantum field theory to super symmetry].
22. Yang CN, Mills RL. Conservation of Isotopic Spin and Isotopic Gauge Invariance, Physical Review. 1954; 96:191-195.